The roles of authority and norm-addressees in deontic puzzles

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Abstract

Attempts to solve classical deontic logic puzzles do not deem relevant the roles of normative authority and norm-addressees. However, drawing a line in imperative logic between the sets of actions validated by authority and the actions that agents can fulfill in different situations prevents apparently unrelated paradoxes from arising at all. This separation also establishes clearer criteria for deeming a result as paradoxical, rather than a vague appeal to intuitions. The old semantic distinction between norm validity and satisfaction is thus brought back and refined through this discussion: norm validity as a reflection of the will of authority; norm satisfaction as the effects of norm-following throughout possible worlds.

Keywords: authority, addressee, validity, satisfaction, agent, puzzle, norm, imperative, deontic.

1 Introduction

Authority and norm-addressees in the logic of norms The will of authority and the actions of norm-addressees are two main actors in the act of norm validation and norm fulfillment or satisfaction. However, this social aspect of the normative phenomenon is usually not deemed as necessary for a basic logic of norms; only propositional contents, the right normative operators and the usual logical connectives are considered to be basic.

It has even been said that the intentions and wishes of authority are not a matter of logic, even if they are necessary to determine whether some norm is correctly satisfying something commanded by authority:

> To determine whether an imperative is 'separable' or 'inseparable', i.e. whether doing A alone produces something 'right' with respect to an im-

> perative $!(A \wedge B)$ or not, it is necessary to examine the intentions and wishes of the authority that used the imperative, it is not a matter of logic. [2, p. 171]

However, the roles of authority and norm-addresses, when they are represented in a formal system through the semantic distinction between validity and satisfaction [1], offer two main reasons to consider the distinction not only relevant, but basic to the logic of norms: first, it solves many apparently unrelated puzzles usually thought to require different approaches to be solved; second, it offers clear grounds to deem a result in normative logic as paradoxical or puzzling, rather than some vague appeal to natural language intuitions. To this end, we propose to take the formal system described in Krister Segerberg's 1990 article 'Validity and satisfaction in imperative logic' [4] as a system formalizing both the logic of norm validation and the logic of norm satisfaction. [2]

One important remark about Segerberg's system is that it is described in terms of imperatives, not in terms of deontic concepts. Nonetheless, we argue that it throws light onto the normative phenomenon in general. We will comment on some of the counterarguments presented by Jörg Hansen in [2]. [3]

We will denote the formal system described in [4] VSL (Validity-Satisfaction Logic). Our purpose is to extend the analysis already offered there; whatever merits are found in the formal system described and the basis of the arguments developed should redirect the reader's attention to Segerberg's numerous works.

Validity and satisfaction in VSL In VSL, Segerberg's goal is to prevent the Ross Paradox, precisely the one which gave rise to this distinction between validity and satisfaction most convincingly. He acknowledges a couple of presuppositions that he needs in order to develop the intended interpretation of his formal system. These presuppositions also describe the main reason why this system represents so adequately the idea that authority and addressees are basic to the logic of norms. The first one has to do with maintaining a separation between the world of facts and the will and actions of authority and norm addressees [4].

> [...] we believe that it is well to leave both the commanding authority (the commander, Ross's "imperator") and the subject (the agent) out of it [the world]. They have different roles to play, both having to do with changes in the world. The subject's is to act; he tries to manipulate the way in which

[1] Long set aside in the discussions on normative logic.

[2] This work takes a standpoint against the idea that we need to choose among truth, validity, satisfaction or some other semantic value for norms, in order to obtain one single true standard deontic logic. Deontic puzzles may require not only a variety of symbolic tools, but also a variety of semantic values.

[3] He puts together important challenges to those who try to develop logics for norm validity and norm satisfaction, but no analysis of the formal system to be described here is presented there, so we will try to answer some of his objections through Segerberg's system and also by refining the notion of satisfaction involved in deontic systems like SDL (Standard Deontic Logic).

[4] He calls them 'subjects' or 'agents'.

> the world changes. The world is in one state one moment, in another state the next; but what the next state is may depend on the subject—on his will. [...] Ultimately it is change in the world that is the authority's concern too, but the ways of authority are indirect, proceeding via the subject. [4, p. 204]

The second presupposition is that the realm of norm validity is seen as a reflection of the will of authority and that this may be achieved by a special semantic device separate from the one describing the different possible situations in a model, which is standard in modal logic semantics. Although he doesn't intend to represent the subject performing any actions because of the complexity of the phenomenon, we suggest there is already a representation of the decisions of agents by way of the actions available to any norm-addressee throughout possible situations, thus taking the context of norm satisfaction as a reflection of norm addressees.

> Here we are doing elementary logic and so shall not be able to do more than scratch the logical surface. In fact, we shall not even touch on the question of how to represent the subject performing any actions. However, we will represent the authority issuing commands. To this end we need to introduce a semantic device to keep track of the commands issued by the authority. [4, p. 204]

The way we are intending to represent authority and norm addressees doesn't allow us to identify and distinguish among separate individual subjects in their role as authority or addressee, but only to distinguish in a most general way among this two different roles. This means it's not relevant if authority is taken to be a singular subject, a group of people, a paper with rules written on it or even the customs of society which eventually may deem some action as normatively valid. It is only relevant to distinguish between the role played by those who validate norms and the very different role played by those who are supposed to follow those valid norms.

2 VSL's language, syntax and semantics

Since VSL is not a well-known system, we will take some pages to describe it, but only certain aspects of it for space reasons, pointing out specially where the notions of validity and satisfaction come up. We will use the same symbols and conventions which Segerberg uses originally, the same names for axioms, inference rules, semantic conditions, language symbols, etc.

2.1 VSL language

Definition 2.1 A *well-formed expression* is either a formula or a term. Every formula is either theoretical or practical, but not both.

(i) Every propositional letter is a theoretical formula.

(ii) $\perp$ is a theoretical formula.
If A and B are formulas:

(iii) $A \to B$ is a formula: practical if A or B are practical, theoretical if both are theoretical.

(iv) $[\alpha]B$ is formula if α is term: it is theoretical if B is theoretical, it is practical if B is practical.

(v) δA is term if A is theorical formula.

If α and β are terms:

(vi) $\alpha + \beta$ is term.

(vii) $\alpha; \beta$ is term.

(viii) $!\alpha$ is practical formula.

(ix) There are no more well-formed expressions.

Theoretical formulas are those true or false formulas which do not contain formulas with imperatives at any point. Practical formulas are those which do contain at some point at least one formula with an imperative.[5] Imperative formulas are considered practical formulas and they don't have truth value, but norm-validity value; they are *prescriptive* norms in the sense that they are valid or invalid on the basis of a defined command set.[6] On the other hand, terms are expressions representative for actions and are of the form 'δA', where A is a theoretical formula and the term is read 'to bring it about that A is the case'.[7] They don't have truth value, neither is it relevant to them the truth value of the formulas which compose them. In this sense they are not actions already *taken* but actions *available* and in order to appear as part of a formula in a truth/falsity context, they need to be part of a modal-dynamic formula as in '[δA]B'. This modal-dynamic formulas '[δA]B' are read as usual in dynamic logic: 'bringing it about that A is the case *leads always to situations where* B is the case', where the 'always' is the reading of the brackets '[ ]'. Again, the complete formal definition of these symbols can be found in Segerberg's article.

2.2 Syntax

Axioms and inference rules The full list of axioms and inference rules can be found in [4, p. 206]. We remark that they are grouped in four categories: propositional, modal, action and imperative.

Propositional

(PA1) *Every instance of tautology of propositional calculus in the VSL language.*

(MP) *If A is a theorem and $A \to B$ is a theorem, then B is a theorem.*

[5] They don't simply describe facts, but depend at some extent on the will of authority.

[6] We could further add a definition for *descriptive* norms, commonly known as normative propositions, also based on this command sets which would have truth value. But it is the main goal of this text to keep things as simple as possible.

[7] There's also an article by Segerberg where he develops this operator [3]

Modal

(MA0) $[\alpha](A \rightarrow B) \rightarrow ([\alpha]A \rightarrow [\alpha]B)$ [8]

(MA1) $[\alpha](B \wedge C) \equiv ([\alpha]B \wedge [\alpha]C)$

(MA2) $[\alpha]\top$

(N) If A is a theorem, then $[\alpha]$A is a theorem, for any α

(MR1) If B≡C is a theorem, then $[\alpha]B \equiv [\alpha]C$ is also a theorem.

Action

(AA1) $[\delta A]A$

(AA2) $[\delta A]B \rightarrow ([\delta B]C \rightarrow [\delta A]C)$

(AA3) $[\alpha + \beta]C \equiv [\alpha]C \wedge [\beta]C$

(AA4) $[\alpha; \beta]C \equiv [\alpha][\beta]C$

(AR1) If A≡B is a theorem, then $[\delta A]C \equiv [\delta B]C$ is also a theorem, given that A and B are theoretical formulas.

Imperative

(IA1) $(!\delta A \wedge !\delta B) \rightarrow !\delta(A \wedge B)$

(IA2) $!(\alpha; \beta) \rightarrow !\alpha$

(IA3) $!(\alpha; \beta) \rightarrow [\alpha]!\beta$

(IA4) $!\alpha \rightarrow ([\alpha]!\beta \rightarrow !(\alpha; \beta))$

(IR1) If $[\alpha]C \equiv [\beta]C$ is a theorem for every C, then $!\alpha \equiv !\beta$ is also a theorem.

2.3 VSL semantics

We offer a brief sketch for the semantics of VSL. Further reference for metasemantic proofs such as soundness and completeness is to be found in [4]. The semantics for VSL are defined in the familiar Kripke structures style, but we first define truth for theoretical formulas and realization for terms in a model. Only later can we define truth or validity for practical formulas.

Definition 2.2 A *VSL model* is a quintuple $\mathfrak{M} = < \mathbf{U}, \mathbf{A}, \mathbf{D}, \mathbf{P}, \mathbf{V} >$, such that:

(i) $\mathbf{U} \neq \emptyset$; **U** is a non-empty set. [9]

(ii) $\mathbf{A} \subseteq \mathbb{P}(\mathbf{U} \times \mathbf{U})$; the elements of **A** are sets of ordered pairs belonging to $\mathbf{U} \times \mathbf{U}$. [10]

(iii) $\mathbf{P} \subseteq \mathbb{P}(\mathbf{U})$; The elements of **P** are *sets* containing elements of **U**. [11]

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[8] This axiom is not stated explicitly in Segerberg's 1990 article, but it is assumed since he acknowledges that this system includes every theorem of the smallest *normal* modal logic [4, p. 211].

[9] Elements of **U** are usually interpreted as possible states or situations where different propositions may be the case.

[10] These are the actions of the model.

[11] We follow the traditional view of characterizing propositions extensionally as the set of

(iv) $\mathbf{D}$:$\mathbf{P}\longrightarrow\mathbf{A}$; $\mathbf{D}$ is a function from $\mathbf{P}$ to $\mathbf{A}$.[12]

(v) $\mathbf{V}$:$\mathbf{L}\longrightarrow \mathbb{P}(\mathbf{U})$; V is the standard *valuation* function.[13]

Each one of this elements should fulfill certain conditions described in [4, pp. 208-9].

2.4 Truth, validity and satisfaction

Truth and satisfaction Truth and satisfaction are defined in [4, p. 209] through a definition of intension for formulas $\|A\|$ and for terms $\| \alpha \|$. We will here focus on the last three (IC5-7), the satisfaction conditions central to the argumentation of this text:

(IC5) $\| \delta A\| = D\|A\|$

(IC6) $\| \alpha + \beta \| = \| \alpha \| \cup \| \beta \|$

(IC7) $\| \alpha ; \beta \| = \| \alpha \| \mid \| \beta \|$

Since each action δA is to be understood as a *set* of ordered pairs, where each pair represents a *transition* between possible worlds or situation, the notion of satisfaction is given in such terms, that is, what satisfies an action is not a state of affairs, but rather a transition between possible situations given by an ordered pair. The definition of satisfaction for terms is thus:

Definition 2.3 Given $\| \alpha \| \in \mathbf{A}$, $< x, y >$ satisfies α if and only if $< x, y > \in \| \alpha \|$.

Such a set $\| \alpha \|$ is defined according to the above definitions (IC5-7), so it ultimately relies on the $\mathbf{D}$ function of the defined VSL-model.

These conditions set the basis for the role of norm-addressees in satisfying the commands issued by authority. In a sense, they offer a way of answering the question: 'What changes in the world would count as performing which actions?', which would be a first step in answerring how to satisfy a certain imperative. The second step would be asking if the action in question is normatively valid, the criteria to answer it comes next.

Norm validity Lastly we get to the definition of norm validity, which is found in the semantics for practical formulas on [4, pp.209-212]. A command system Γ in a VSL-model $\mathfrak{M}$ is defined as:

$\Gamma = \{\Gamma_x : x \in \mathbf{U}\}$

where Γ_x is called a command set of x. Any command set Σ should fulfill conditions (C0-4) found in [4, p. 210]. We will emphasize only the first two:

(C0) $\Sigma \subseteq \mathbb{P}(\mathbf{U} \times \mathbf{U})$

(C1) if $DX, DY \in \Sigma$ then $D(X \cap Y) \in \Sigma$, for all $X, Y \in \mathbf{P}$.

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situations or states where they are true.

[12] This is the δ-operator, which defines an action in terms of a transition that leads to a certain proposition being true.

[13] $\mathbf{L}$ is a set of propositional letters in VSL's language. Thus the function assigns a set of elements of $\mathbf{U}$ to each propositional letter from the language of VSL.

Condition (C0) is central since it states that command sets are given in the same terms as actions; as sets of ordered pairs of elements of **U**, sets which represent transitions between possible situations, so that only transitions are commanded and not directly propositions. Condition (C1) will be refered to when analyzing the paradox of conflicting oblgiations in the next section. It should also be noted that the conditions (C0-4) make it possible that there are empty command sets.

Norm validity is thus defined by recursion in (RC1-5) [4, p. 211]. We will only cite the most relevant one:

(RC5) $\Gamma \models_x !\alpha$ iff $\| \alpha \| \in \Gamma_x$.

The crucial definition (RC5) tells us that a commanded term is normatively valid whenever it is found in the command system of the world in question. This represents what is commanded by authority.

Other definitions We present important definitions not explicit in Segerberg's article.

Definition 2.4 A formula ϕ is *VSL-valid* if at any $x \in \mathbf{U}$ of any $\mathfrak{M}$, if ϕ is either true or normatively valid in x. That is, if either $x \in \| \phi \|$ or $\Gamma \models_x \phi$.

Definition 2.5 A formula ϕ is *logical consequence* of ξ, if there is no model $\mathfrak{M}$ and a situation $x \in \mathbf{U}$ of the model where: ϕ is either true or normatively valid in x and ξ is neither true nor normatively valid in x.

3 Reinterpreting three classic puzzles

We will discuss three main puzzles: the logical necessity of obligations, the paradox of conflicting obligations and Chisholm's paradox. The reasons for these particular choices are mainly to show how paradoxes with different formal sources may be approached through this lens. [14]

3.1 Logical Necessity of Obligations

This puzzle is a clash between a very intuitive idea about the contingency of norms and a very straightforward result of SDL (Standard Deontic Logic). The idea of contingency of norms may be expressed in a simple statement:

(1) There are possible worlds without norms.

The theorem of SDL in clash with (1) is:

(2) (ON) $O\top$ [15]

This theorem basically says that it is a logical truth that every tautology is obligatory. Since being a logical truth means being true at all posible worlds, it follows that in all possible worlds tautologies are obligatory, so none of those worlds is without norms.

[14] This analysis is part of a PhD thesis to be presented in the Autumn of 2020, under the tentative title: 'Validity and satisfaction in deontic logic', by the first author of this text. Eight more puzzles are analyzed in that text.

[15] Reading '$\top$' as any tautology of propositional logic.

It is no mystery that the source of (ON) is the necessitation rule together with the tautologies of propositional calculus. But why is it deemed as paradoxical? What does it mean to say that according to this theorem 'tautologies *are* obligatory' and to say that 'there *are* possible worlds *without* norms'? Under what standard should we interpret there *being* norms or *not being* any norms. We should review the semantic definition of formulas with the form $\mathrm{O}\alpha$ in SDL.

The semantic conditions of truth in terms of Kripke semantics state that, for a formula Op to be true in a world x, the formula p has to be true in all the worlds which have the normative-acceptability relation with the world x. Given this definition, it should be obvious that the theorem has to be a logical truth, since tautologies are true in all possible worlds, no exception for the normatively acceptable ones. Thus the formula is satisfied throughout normatively acceptable possible worlds, it's modally-normatively satisfied.

But why then is the result paradoxical? It would be hard to argue against the possibility of worlds without complex beings, complex enough to state rules. Our own universe didn't have any normative authorities when life wasn't even possible, in that possible world there would be no norms. The crucial focus here is on the word *are* when saying 'there *are* no norms'. What do we mean by that? The most natural answer may be that norms haven't been stated or validated by anyone.[16] In that sense, of course, there are possible worlds where no norms have been validated, in the abscence of any normative authority necessary for that action.

SDL clearly fails to follow that simple intuition, but it correctly expresses a truth about the context of satisfaction[17] of the norm-validating and norm-following activities. Namely, that the criteria to consider an event as going according to some norm is trivially satisfied even in the abscence of conscious agents capable of even understanding rules.[18] In this sense, from the viewpoint of norm-addressees, this theorem says that they shouldn't worry about obligations of making a tautology true, for anything they do or don't do will trivially satisfy that obligation. This may be the first intuitive answer one may think about when first trying to make sense of this puzzle.

VSL doesn't have this ambiguity problem, for it has separate criteria to deem a normative formula as valid and a theoretical formula as true or satisfied.

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[16] In this discussion it should be prefered to talk about 'validating' a norm, instead of 'stating' one, for 'stating' may be interpreted as'uttering', which is not necessarily meant when talking about norm validity. A norm may be uttered and thus validated, but its logical consequences may have not been uttered and still be validated by their antecesor.

[17] The term 'context of satisfaction' is taken from a very interesting account of this concept in [5]

[18] This is argued in the same spirit in which propositions are said to be true even in possible worlds where there are no living creatures capable of uttering any sentence. Since uttering a true sentence and its being true can be considered different matters, commanding an imperative and its being satisfied can also be considered different matters. An event may occur which would make some sentence true, even if that sentence has never been uttered. In the same way, an event may occur which would count as satisfying some command, even if that command has never been uttered.

That is, the theorem is not logically valid in VSL, but the fact that anything would count as satisfying such a command is also preserved. This is how this same theorem would look in VSL:

(3)$(VON)!\delta\top$

This formula (VON) should be interpreted as saying:

(4) Bring it about that $\top$ (an instance of any tautology) is the case!

Clearly, any action or even any failure to act would satisfy such a command, in every possible world. There are only two conditions which the function **D** has to fulfill whenever it belongs to a model $\mathfrak{M}$ of VSL, given in [4, p. 209] as FD1 and FD2. These should hold for any X, Y$\in$**P**

(FD1) $DX \subseteq \{< x, y >: y \in X\}$

(FD2) If $< x, y >\in DX \Rightarrow y \in Y$, for all y, then $< x, z >\in$DX$\Rightarrow< x, z >\in DY$.

The relevant condition for this puzzle is clearly (FD1), since it states that the *image* of the function assigned to a proposition through the δ-operation should be contained in the set of worlds represented by the proposition affected by the operation. In other words, 'doing A' should always get you to a state where 'A' is the case. Since $\top$ is the case in every possible world, any set of ordered pairs $\parallel \alpha \parallel\in$**A** will count as seeing to it that $\top$ is true, for any set of ordered pairs will take us to worlds where $\top$ is true. In other words, the way we defined what counts as doing a certain action makes it true that any action whatsoever will count as seeing to it that a tautology is true.

But that wouldn't be enough to deem (VON) as logically valid in VSL, for the semantics of formulas with the form $!\delta A$ are not based on what propositions are true in any possible world or situation of the model, but rather on command sets specific to each possible world. These command sets may be empty according to the semantic conditions in [4, pp. 209-211]; in that case $\parallel \delta\top \parallel$ wouldn't belong to the command set in question, thus it wouldn't be normatively valid in virtue of mere logic. Given (RC5), it would be necessary to have the specific formula (18) in a command set of a certain world to deem it as valid, regardless of the fact that any action would count as $\delta\top$, thus distinguishing clearly between what norms agents fulfill by doing '$\delta\top$' (anything at all) and what the authority wills to be done (perhaps not particulary $\delta\top$).

3.2 Paradox of conflicting obligations

This paradox calls for attention to the intolerance of standard deontic systems towards normative conflict, making it escalate to a logical contradiction. Consider thus this two conflicting obligations:

(1) Op

(2) O¬p

It's not hard to prove that from (1) and (2) a contradiction is derivable in SDL. It's also commonly accepted that even a conflict of obligations as impossible to fulfill simultaneously as the one between (1) and (2) shouldn't be considered as serious and strong as a logical contradiction.

But we could ask again exactly what aspect of norms makes it so paradoxical that a conflict of norms derives in a logical contradiction. Let's introduce the points of view of authority and norm-addressees to clear out this question.

From the point of view of norm-addressees and specifically when considering the context of satisfaction of (1) and (2), trying to fulfill both obligations would amount to a contradiction. The very semantic conditions of SDL make it clear that making both true in a certain world would require that in all normatively adequate worlds both p and ¬p be true, which would cause contradictions in all such worlds. This supports the idea that a certain notion of satisfaction is the best way to interpretic deontic formulas in SDL.

From the point of view of authority, there's certainly some kind of tension (maybe a *rationality* tension) between two norms which can't be fulfilled, but it's not clear that this tension should amount to a contradiction. This point of view favojrs the idea which gives rise to the paradox: a conflict of obligations is not as strong as a logical contradiction.

But let's consider how VSL deals with this sort of conflict. The traslation of (1) and (2) would be:

(3) $!\delta$A

(4) $!\delta\neg$A

Where A is any theoretical formula of VSL. There should be no doubt that the normative conflict between (3) and (4), although stated in terms of imperatives, is completely analogous to that of (1) and (2). Does VSL allow a logical contradiction from (3) and (4)? The short answer is no, but the details are very interesting: although (3) and (4) don't result in a logical contradiction, they can't be both normatively valid in the same world; also, the system allows us to see that both commands can't be simultaneously fulfilled, so the system tells us something about the context of validity and also about the context of satisfaction of these commands in VSL.

Firstly, these formulas don't result in contradiction because these are imperatives, practical formulas which have no relevance in the factic description of any world of a VSL-model, this task is left to theoretical formulas. Axioms and rules of VSL back up this, since none of them is analogous to the axiom of SDL which gives rise to the paradox: Op$\rightarrow$ ¬O¬p.

Secondly, one consequence of rendering both formulas normatively valid is a trivialization of the norm-giving activity of authority. This is due to semantic condition for command sets C1. Given (C1), if both (3) and (4) belong to a command set, the following should also belong to that command set:

(5) $!\delta$(A$\wedge\neg$A)

This would amount to command an impossibility, just as in SDL Op and O¬p would amount to commanding an impossibility. This can happen in VSL according to its semantic conditions, not only can a command set be empty, but also the empty set could belong to a command set. However, the problem doesn't escalate to a logical contradiction, it just makes the command system of the model useless, as Segerberg proves in his Proposition 4.1 [4, p. 210]. This proposition says that, if $\emptyset \in \Gamma_x$, then Γ_x=**A** and if $< x, y >\in$R, for any R$\in$**A**, then Γ_y=**A**. That is, it deems the command set, and all the command sets with which it is related through an action, equal to the action set of the model, so everything would be normatively validated. But this is different from having a propositional contradiction, for this trivialization of norm validity would not make every theoretical formula a theorem. The system is at least able to contain the explosivity of normative contradiction within the domain of normative reasoning, without affecting the descriptive or propositional part of the system.

This view is in consonance with the intuition that there is some tension between (3) and (4) which should neither scalate to contradiction, nor should it be normatively or rationally indifferent for authority to command contradicting things.

Lastly, we notice that commands (3) and (4) could not possibly be *fulfilled* simultaneously. Because of the way the δ-operator is defined, there is no way to execute both δA and $\delta\neg$A, for it would require a world where A and $\neg$A were true. [19] Norm-addressees would certainly backup the idea that fulfilling both commands would amount to a logical contradiction.

3.3 Chisholm's paradox

This paradox can't be solved as straightforwardly as the last two and that is why we chose it for exposition, to show some of the drawbacks of VSL and the distinction in question. Notwithstanding, some aspects of the paradox are very interesting under this light.

This paradox is usually taken to reveal SDL's lack of capacity to represent normative conditionals and also its questionable capacity to deal with normatively unacceptable worlds, where a logic of norms should nevertheless hold. [20] As we will see, VSL's modal-dynamic symbols add some expressive capacity in those areas, but it remains questionable how succesful it is in solving this puzzle since it also requires an additional semantic condition and an additional axiom to represent some important intuitions about this sentences.

Consider the following statements: [21]

(1) It ought to be that John goes to the assistance of his neighbours.

(2) It ought to be that if John goes to the assistance of his neighbours, then he tells them he is coming.

[19] But this leaves the possibility to fulfill both commands sequentially, first getting to a world where A is the case and then to one where it is not the case or viceversa: (δA; $\delta\neg$A) or ($\delta\neg$A; δA).

[20] This already hints to the relevance of distinguishing between validity and other semantic values, it leads to evaluate separately which norms have been violated or fulfilled and which norms are nevertheless valid or invalid.

[21] The formulation is taken from [1, p. 83]

(3) If John doesn't go to the assistance of his neighbours, then he ought not tell them he is coming.

(4) John does not go to their assistance.

The problem arises when different formalizations of this four sentences clash with at least one of the following intuitions about them: That (1)-(4) are mutually consistent and also logically independent.

The formalizations differ on the scope of the normative operator in sentences (2) and (3), precisely the sentences with conditionals involved. They can be symbolized with a wide scope (as in '$O(p \rightarrow q)$') or with a narrow scope (as in '$p \rightarrow Oq$'). Either way, and no matter which of the conditionals is symbolized in these different ways, the set of sentences will be either inconsistent or there will be logical dependency among some of the sentences.

Let's see how VSL could symbolize this set of sentences without using the classical conditional:

(5) $!\delta A$

(6) $[\delta A]!\delta D$

(7) $[\delta \neg A]!\delta \neg D$

(8) $\neg A$

The modal-dynamic ingredient added in VSL allows us to represent conditionals (2) and (3) in terms the worlds or situations where doing A or not doing A would take us (whether John fulfills or doesn't fulfill his obligation stated in (1)). This is one of the two important aspects where SDL fails, in its capacity to deal with less-than-ideal worlds, meanwhile VSL can tell us which command is valid in any of the two cases, even considering that (5) validates that δA is the normatively right way to go.

From the point of view of authority, this sentences are covering what authority wills to be done, whether the addressee decides to assist his neighbors or not: the commands that would hold as valid in any of the two cases are already foreseen by the command sets representing the will of authority. From the point of view of John, the norm-addressee, it is also clear which actions would satisfy which commands and the consequences of his choices.

Regarding the alleged consistency and independence of this sentences, both seem to be respected. It is to be noted that this particular modeling in VSL allows an inference when considering axiom AI4[22]:

(AI4) $!\alpha \rightarrow ([\alpha]!\beta \rightarrow !(\alpha;\beta))$

The inference seems harmless, since all we get is $!(\delta A;\delta D)$. But it does reveal that the modal-dynamic brackets may not be rescuing the conditional spirit of the advice given in (6), for the result would read something like this: 'Assist your neighbors! Then tell them you are coming!', which is certainly bizarre since it shouldn't be a matter of timing or order of execution, it should

[22] [4, p. 206]

rather be a matter of a command being valid whenever some action should be taken. But (6) describes what happens when the action is taken, not when it is normatively valid; therefore, that couldn't be represent either, since the rules for well-formed expression only allow terms to be between modal-dynamic brackets, not imperatives or formulas of any kind.

We could stick to the traditional implication, leaving (5) and (8) as they are and changing only (6) and (7):

(6') A→!δD

(7') ¬A→!δ¬D

We would be following the narrow scope formalization, but then we could derive (6') from (8), using propositional logic and thus would be rendering the sentences not logically independent.

Another observation is that, if we stick to the formalization with brackets, there's yet another problem regarding (7) and (8). That is, it should be clear that from the original sentences (3) and (4) follows that John ought not tell his neighbors that he is coming, which is precisely the consequent in (3). But from (7) and (8) VSL doesn't allow us to infer the validity of '!δ¬D'. For this, an additional action axiom and an additional semantic condition for the function **D** would be needed:

(AA5) A→([δA]B→B)

(FD3) For any u∈**U** and X∈**P**, if u∈X, then $< u, u >$∈DX

The axiom means that if you are in a world where A is the case and doing A always takes you to a situation where B is the case, then you are already in a situation where B is the case. [23] . The condition FD3 reflects this on a semantic level by saying that any world where a proposition X holds should always be included as one of the possible transitions that leads to X. [24]

Further observations could be made, but the goal of showing the limitations of this distinction between authority and norm-addressees to solve this puzzle has already been met.

4 Counterarguments and further questions

Valdity as utterance An interesting argumentation line against the very idea of a logic of imperative validity is that, in trying to explain what it means to logically infer one imperative from other, we may be implying that the *existence* of an imperative somehow implies the *existence* of another imperative. Hansen cites [2, p. 153] many examples of different logicians warning us against the idea that a logic of imperative may be understood as the logic of the *existence*

[23] This may lead to say that whenever something is the case you may assume some action lead to it, which is rather questionable since actions are usually thought of in relation to some agent. It would be a matter of discussion how this affects the characterization of action in this system

[24] This is also an odd condition, since it is doubtful that any instance of $< u, u >$ should really count as a *transition* between situations.

of a command, its utterance, the action of stating or similar definitions. The warning is certainly helpful, but it hardly undermines the very idea of a logic of norm validity. Let us take the example from Aleksander Peczenik:

> The premiss 'love your neighbour' may be regarded as describing the fact that the authority – Jesus – has in fact said 'love your neighbor'. The imperative existed because it was uttered by Jesus. But the conclusion, for example, 'love Mr. X' does not describe anything which in fact has been said by Jesus. [25]

In this sense, to say that an imperative exists would mean something like the 'stating' or 'uttering' of a certain command in the right context. But why should we define norm validity like that? In VSL, for example, there are conditions to ensure some kind of rationality for command sets, the ones listed (C0-4). Condition C1, for example, says that if two different commands belong to the command set, then the **D** function of the intersection of both propositions also belong to the set, thus rendering it normatively valid. That the set of commands issued by an authority may be closed under logical consequence doesn't mean that authority is somehow 'silently uttering' commands, but rather that authority is committed to the validity of implicit norms, just as asserting propositions may committ a speaker to the implicit truths.

In general, we may take this kind of objection as a healthy warning against the identification between the kind of actions that may render some norm as valid (its utterance, for example), and the validity itself, which may be acquired in a variety of ways (by being written in some particular place, being uttered by some person, being the result of a certain social convention, being performed by a number of individuals through a long period of time, being implied by some other valid norm, etc.). Taking the warning seriously doesn't require to throw away altogether the concept of a logic of norm validity.

Imperatives and deontic concepts A more serious objection may be that it is doubtful that a logic of imperatives can contain a logic of deontic concepts without loss. Even by defining the obligation operator O exactly as the imperative ! in VSL, it is doubtful that the specific logic for the concept of permission would be adequately represented. This would affect the scope of the present arguments, so that we would only be arguing for the relevance of the distinction between authority and norm-addressees regarding the logic of imperatives and not for the logic of norms in general.

Moreover, an interesting problem arises when trying to define the concept of permission in terms of the distinction between validity and satisfaction. When the propositional content of an obligation is true, it's satisfied; when it's false, it's violated. If I have an obligation to pay taxes, then whenever it is true that I pay them, the obligation is certainly satisfied. The same may hold for imperatives, but it doesn't hold for permission, for if I have a permission to drive a car, were it true that I drive one doesn't satisfy it in any meaningful

[25] In a letter by Peczenik found in [6], cited in [2, p. 152]

sense and were it false, it wouldn't be in violation of the permission.

What, then, is to satisfy a permission? A possible answer may lie in comparing permissions to rights, in the sense that they are duties or *obligations* for authorities (usually norm-givers), thus inverting the directionality of responsibilty for norm satisfaction from norm-adreessees to norm-givers or authorities.

5 Conclusions

Although Chisholm's Paradox couldn't be solved with the aid of the distinction between authority and norm-addressees, the other two paradoxes were not only overcome, but it is clearer why they are considered as paradoxes to begin with: it depends on the semantic values we are assuming to evaluate the soundness of each result, either validity or satisfaction. This is where we intend to make a contribution, in showing that not only the Ross paradox calls for an analysis which makes a clear semantic distinction between norm validity and satisfacion, but other classical paradoxes could also be better understood and solved through this approach.

We also wish to address how the semantic distinction in question is also related to the points of view of authority and norm-addressees, suggesting that their relevance to a standard logic of norms may be more important than usually regarded. The system VSL may be too complex to be considered basic to the logic of norms, but its capacity to solve paradoxes calls for a detailed analysis of the aspects of this complexity that should be preserved in order to define a truly standard logic for normative reasoning.

The suggestion to add axiom (AA5) and semantic condition (FD3) is a small formal contribution to VSL. Hopefully, it adds to the suggestions of possible ways to make more positive contributions to the enrichment of systems of normative logic seeking resources from dynamic logic to solve the problems of conditionals in normative contexts and the logic of norms in less-than-ideal situations.

The correct interpretation of deontic formulas in standard systems such as SDL is problematic, but a refinement in the notion of norm satisfaction may help reivindicate the line of interpretation which leans towards *satisfaction* for this systems. The refinement here proposed would be the one we called 'normative-modal satisfaction' in Section 3.1: the fulfillment of a norm in a specific set of possible worlds and not just its truth value in the actual world.

The problem of defining a reasonable notion of satisfaction for permission in terms of inverting the directionality of responsibility among authority and norm-addressees was also brought to light through this view. It could also be argued that the notion of satisfaction can't make sense at all regarding the concept of permission, but it would be hard to explain the relevance of permissions in evaluating the overall fulfillment of a normative system which includes permissions.

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